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# Separate, but Better? Measuring School Spending Progressivity and its Association with School Segregation

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Recent public discussions and legal decisions suggest that school segregation will remain persistent in the United States, but increased transparency may help monitor spending across schools. These circumstances revive an old question: is it possible to achieve an educational system that is separate but equal—or better—in terms of spending? This question motivates further understanding the measurement of spending progressivity and its association with segregation. Focusing on economic disadvantage, we compare two commonly-used measures of spending progressivity: exposure-based and slope-based. We show that each measure is predicated on different assumptions about the progressivity of within-school resource allocations, and that they are theoretically linked through segregation. We empirically examine school spending progressivity and its properties using nationwide school spending data from the 2018-19 school year. Consistent with our theory, the exposure-based measure is the slope-based measure shrunk inversely by economic school segregation. This property makes more segregated school districts look more progressive on the exposure-based measure, representing a seemingly "separate but better" relationship. However, we show that this provocative pattern may be reversed by relatively modest poor-versus-nonpoor differences in unobserved parental contributions. We discuss implications for the measurement of progressivity, and for theory on public educational investments broadly.

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# Separate, but Better?

## Measuring School Spending Progressivity and its Association with School Segregation

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## Abstract

Recent public discussions and legal decisions suggest that school segregation will remain persistent in the United States, but increased transparency may help monitor spending across schools. These circumstances revive an old question: is it possible to achieve an educational system that is separate but equal—or better—in terms of spending? This question motivates further understanding the measurement of spending progressivity and its association with segregation. Focusing on economic disadvantage, we compare two commonly-used measures of spending progressivity: exposure-based and slope-based. We show that each measure is predicated on different assumptions about the progressivity of within-school resource allocations, and that they are theoretically linked through segregation. We empirically examine school spending progressivity and its properties using nationwide school spending data from the 2018-19 school year. Consistent with our theory, the exposure-based measure is the slope-based measure shrunk inversely by economic school segregation. This property makes more segregated school districts look more progressive on the exposure-based measure, representing a seemingly "separate but better" relationship. However, we show that this provocative pattern may be reversed by relatively modest poor-versus-nonpoor differences in unobserved parental contributions. We discuss implications for the measurement of progressivity, and for theory on public educational investments broadly.

**Keywords:** progressivity, school finance, school segregation

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## 1 Introduction

Desegregation and the allocation of school resources have been the key policy levers to address the "separate and unequal" K-12 education system in the United States. While at least explicit *de jure* segregation has been stricken out from the system since the *Brown* decision (1954), the willingness and ability of governments at all levels to force *de facto* school desegregation have waned substantially after a series of court decisions (Reardon et al. 2012). In addition, the proliferation of public school choice has provided the advantaged with options to avoid attending schools with the disadvantaged (Candipan 2019; Rich, Candipan, and Owens 2021). Consequently, U.S. public schools today are highly segregated by both race/ethnicity and economic status, with growing trends in economic school segregation (Owens et al. 2022).

While desegregation has become more difficult to achieve, several aspects of the public education system have changed. First, since the 1970s, state-level school finance reforms (SFRs) and legislative policy have aggressively increased the funding for school districts serving more disadvantaged students (Rothbart 2020; Oberfield and Baker 2022). Second, the demand for educational accountability and transparency has risen, which made school- and district-level finance (revenues and expenditures) data publicly available. These changes suggest that it may finally be possible to hold governments accountable to the "separate but equal" doctrine where, despite the segregated school system, the schools serving the disadvantaged are at least as well resourced, and possibly even more resourced, than the schools serving the advantaged.

Formal and informal (e.g., public shaming of governments) accountability for equity in educational finance requires monitoring the equity of educational resources experienced by students of different backgrounds (Baker et al. 2021; Blagg, Gutierrez, and Terrones 2022). In an equitable system, disadvantaged students should experience schooling with no less, and possibly

more, educational resources than advantaged students. The extent to which this is achieved is termed the *progressivity* of school spending, an indicator of equity in educational expenditure across students by disadvantage. In this paper, we focus on poverty as our measure of disadvantage and study whether more public expenditures were spent on poor students' education than non-poor students', a pattern we call progressive. A regressive school finance system has the opposite relationship, and a flat system would equalize spending.

Two particular measures have been widely used to evaluate the progressivity of school spending. The first, which we call the slope-based measure of progressivity, describes the linear relationship between the share of poor students and per pupil spending. The second, which we call the exposure-based measure, indicates the difference in school-level per-pupil spending for the average poor versus non-poor students. As we detail in this paper, the slope-based measure relies on an assumption of systematic within-school resource targeting (e.g., within-school tracking policies that separate poor and non-poor students into classrooms with different resources), while it may better capture the underlying parameters of school finance policy. On the other hand, the exposure-based measure captures not only the underlying school funding policy but will directly reflect observed segregation. This latter measure is justified insofar as resources are distributed equally within schools ("within-school equality"), which may better reflect *de facto* progressivity as experienced by students.

Using recent nationwide school-level finance data, we find that public spending in the U.S. education system is progressive on average. Moreover, consistent with our theory, the exposure-based measure of progressivity is shrunken from the slope-based one in proportion to economic segregation, so the measure is shrunk less among more segregated districts. Consequently, the exposure-based measure presents a provocative "separate but better" pattern,

in which more segregated districts (compared to less segregated districts) spend relatively more of their public educational resources on poor students than non-poor students. On the other hand, the slope-based measure presents a different pattern in that more segregated districts are not substantially more progressive in educational spending than less segregated districts.

One important caveat in the present study—as with many other educational finance and policy studies—is that it only includes reported public expenditure and does not account for unobserved parental contributions. We discuss how the "separate but better" pattern observed for the exposure-based measure can be undone if we adjust for parental contributions with the assumption that non-poor students bring more unobserved resources to their public school than poor students (whether through direct donations, volunteering, or peer effects). A parental contribution gap (non-poor minus poor) of about $1,750 per student per year is sufficient to return us to a "separate and unequal" pattern. This is not an unreasonably large gap based on the available literature, but we encourage future research to more carefully address this critical consideration by better quantifying reasonable estimates of the parental contribution gap.

The remainder of the paper is organized as follows. In Section 2, we formalize the exposure-based and slope-based measures of progressivity in school spending. We introduce "student-based budgeting," a simplification of school finance policy, and derive a key theoretical relationship between these measures through the exposure-based segregation measure. We also show how to decompose each measure into within- and between-district components that can reveal the direction and contribution of each component to the overall progressivity within states. Section 3 discusses the conditions under which each measure of progressivity accurately describes the gap between poor and non-poor students in educational spending. Section 4 introduces the datasets used in this paper's empirical contribution. Section 5 describes school

spending progressivity in the U.S. and its decomposition into within- and between-district components. Section 6 examines the relationship between the two measures of progressivity and economic segregation. We also show how the association between segregation and progressivity can appear differently depending on assumptions about unobserved parental contributions. Section 7 concludes.

## 2 Two Measures of Spending Progressivity

Two measures of school spending progressivity have been commonly used in the school finance literature: exposure-based and slope-based progressivity. We introduce these measures in this section, and discuss how each of these measures has advantages and disadvantages.

We denote the exposure-based measure of progressivity by $\text{Prog}_E$ and the slope-based measure by $\text{Prog}_S$. The definition of $\text{Prog}_E$ is a difference in weighted means, specifically the difference in school spending between the average poor and non-poor student's school. Formally, let $N_{ps}$ (or $N_{ns}$) and $N_p$ (or $N_n$) be the number of poor (or non-poor) students in school $s$ and in the entire state $t$ (or region of study), respectively, and $PPEXP_s$ be expenditure per pupil in school $s$. Then the progressivity of school spending is

$$\begin{aligned}\text{Prog}_E = &\text{ (per-pupil spending in the average poor student's school)} \\ &-\text{(per-pupil spending in the average non-poor student's school)} \\ = &\sum_s \frac{N_{ps}}{N_p} PPEXP_s - \sum_s \frac{N_{ns}}{N_n} PPEXP_s = \sum_s \left(\frac{N_{ps}}{N_p} - \frac{N_{ns}}{N_n}\right) PPEXP_s \end{aligned} \quad (1)$$

This exposure measure of progressivity has been used in several research studies such as Sosina and Weathers (2019) and Shores, Lee, and Williams (2021), and is perhaps most famously the progressivity measure used by the Urban Institute in their interactive website,

"Which Students Receive a Greater Share of School Funding?" (Chingos and Blagg 2017; Blagg, Gutierrez, and Terrones 2022), although the Urban Institute takes a ratio of the weighted means, rather than the difference.

The slope-based measure of progressivity, $\text{Prog}_S$, is the linear relationship between school per-pupil spending and the share poor in the school. This measure is commonly observed in scatterplots since that graphical depiction makes it natural to add lines or curves of best fit. We define slope-based progressivity as the bivariate (linear) regression slope,

$$\text{Prog}_S = \frac{Cov(EXP_s, p_s)}{Var(p_s)}$$

Where $p_s$ denotes school poverty rates in school $s$, $Cov(\cdot,\cdot)$ denotes covariance, and $Var(\cdot)$ denotes variance. The slope measure has been used perhaps most famously in Baker et al. (2021). They adjust this slope measure for other district-level observables, but fundamentally their measure is based on the regression slope. Other studies that use this measure include Lane, Linden, and Stange (2018) and Farrie, Kim, and Sciarra (2019).

The slope-based progressivity measure may be susceptible to measurement error under the following circumstances. First, the slope measure becomes very noisy to estimate in jurisdictions with little segregation. If all the institutions have the same rate of poverty, there is no way to measure the slope relating per-pupil spending to poverty. Yet the exposure measure of progressivity can be easily measured in such cases, and it is approximately zero (poor and rich students are exposed to the same levels of school-level spending under no income segregation). Moreover, the slope measure may not be applicable to districts with no schools having extremely high or low poverty rates since it relies on extrapolation. For example, suppose we measure progressivity at the level of the school district. If school districts do not have any schools with very high shares of poor students, the slope measure will be based only on schools with low

shares of poor students. The extrapolation in this case makes it unclear whether these school districts would, in fact, spend the predicted per-pupil amount in schools of very high rates of poverty. For example, 1,336 out of the 11,527 school districts with at least 2 schools do not have any schools with at least 30% free or reduced-price lunch eligible students.

The exposure-based measure is intuitive and feasible to estimate even in places with low variation in poverty rates across schools. The chief "disadvantage" of this measure lies in the implicit assumption it makes about the within-school distribution of spending. This implicit assumption is closely related to the perhaps surprising relationship this measure has with segregation. However, these "disadvantages" may actually be useful features of the measure, depending on whether the assumptions underlying them are true. This discussion will be clarified in the next subsection and thereafter.

### 2.1 Student-Based Budgeting Model

In this paper, we use the term "student-based budgeting model" to refer to a school spending system where the total spending (or budget) allocated to a school is a linear function of student enrollments by type. This model is not directly reflective of how spending is allocated in practice, although this approach has been directly adopted in specific states and large school districts (Roza, Hagan, and Anderson 2021) and is often advocated as a means to improve spending efficiency and equity (Jarmolowski, Aldeman, and Roza 2022). Therefore, we use this model as an approximation to the far more complicated actual spending regime that considers diverse programs and requirements. Starting with the assumption of student-based budgeting permits us to transparently derive a theoretical link between the two measures of progressivity that operates through segregation.

Since the focus of this paper is spending progressivity in relation to economic

disadvantage, we will focus on the share poor as the student-body characteristic of interest. We call budgeting at school $s$ "student-based" if the total budget allocated to $s$ is a linear function of student enrollments by characteristics. Equating budget with school spending yields the equation,

$$EXP_s = \alpha_f + \alpha_p N_{ps} + \alpha_n N_{ns}$$

where $EXP_s$, $N_{ps}$, and $N_{ns}$ were defined earlier, $\alpha_f$ are the fixed allocations per school, $\alpha_p$ is the amount of money allocated per poor student, and $\alpha_n$ the amount per non-poor student. Dividing by the total number of students in the school, $N_s = N_{ps} + N_{ns}$ and defining $p_s = \frac{P_s}{N_s}$, as the share poor, and $PPEXP_s = \frac{EXP_s}{N_s}$ as the spending per-pupil, yields the per-pupil equation for student-based budgeting

$$PPEXP_s = \alpha_0 + (\alpha_p - \alpha_n)p_s = \alpha_0 + \text{Prog}_S \times p_s$$

where $\alpha_0$ is the intercept. We have substituted $\text{Prog}_S$ in the last expression since if budgeting is student-based (and only based on share poor) then there will be an exact linear relationship between per-pupil expenditure and share poor, consequently $\text{Prog}_S$, which was defined as the bivariate linear regression slope, will be exactly the difference in policy parameters $\alpha_p - \alpha_n$.

Student-based budgeting is a fictitious and simplified model of resource allocation. The process allocating resources to schools is very complicated. Some of the violations of the model are immediate: several state governments allocate educational resources not purely linearly based on student characteristics, but also add provide additional funds to places of concentrated disadvantage (Fischer, Duncombe, and Syverson 2021). Such a policy makes the budgeting model explicitly nonlinear (convex in many cases), violating the student-based budgeting form. Moreover, not all revenues are allocated by states, and so the resource allocation rule will depend

on local tax revenues and local voting, in addition to Title I funds, which are allocated to schools serving poor students based on a cutoff rule (van der Klaauw 2008; Matsudaira, Hosek, and Walsh 2012) and therefore, nonlinearly.

However, despite being simplified, student-based budgeting is a useful conceptual model for resource allocation. It is the simplest model of resource allocation that yields a progressivity policy parameter, specifically the slope on percent poor in the school (i.e., $\text{Prog}_S = \alpha_p - \alpha_n$). It is important to note that this "policy parameter" is not controlled by any one government in practice; consequently, it could be viewed as a policy outcome rather than a policy choice. But we will still refer to it as a policy parameter, given that it plays the role as a parameter in our conceptual model. If we believe that this model is approximately true, we can recover this policy parameter from the data, specifically by calculating the slope relating school-level per-pupil expenditure $PPEXP_s$ with school-level per-pupil rates of poverty $p_s$. As shown, this parameter is exactly $\text{Prog}_S$, the slope-based progressivity measure. This leads to Observation 1.

**Observation 1:** If the educational resource allocation policy is student-based budgeting based on student poverty, then the slope-based measure of progressivity is exactly the parameter of the student-based budgeting policy.

We introduced student-based budgeting at the school-level, $s$. However, student-based budgeting could also apply at the district-level, or within schools across classrooms (e.g., through program participation or tracking). If student-based budgeting applies with the same weights at every institutional level, then the slope progressivity measure is invariant to aggregation. This leads to Observation 2.

**Observation 2:** If the educational resource allocation policy is student-based budgeting based on student poverty, and every level of government uses the same allocation weights, then the school

funding slope measure of progressivity is invariant to the level of aggregation.

## 2.2 Progressivity and Segregation

Under the assumption of student-based budgeting, we can derive an important equation relating the two progressivity measures that operates through segregation. First, we define the traditional exposure-based economic segregation measure (Jang 2022) $\text{Seg}_E$ as

$$\text{Seg}_E = \sum_s \left( \frac{N_{ps}}{N_p} - \frac{N_{ns}}{N_n} \right) p_s$$

Where $N_p$ (or $N_n$) is the total number of poor (or non-poor) students, $N_{ps}$ (or $N_{ns}$) is the number of poor (or non-poor) students in school $s$, and $p_s$ is the proportion of poor students in school $s$. This is the poverty rates in the average poor student's school, minus the poverty rates in the average non-poor student's school, which is sometimes referred to as the variance ratio index (James and Taeuber 1985). Proposition 1 is our main theoretical result.

**Proposition 1:** If the educational resource allocation policy is student-based budgeting based on student poverty, then

$$\text{Prog}_E = \text{Prog}_S \times \text{Seg}_E$$

**Proof:** By assumption,

$$PPEXP_s = \alpha_0 + (\alpha_p - \alpha_n) p_s = \alpha_0 + \text{Prog}_S \times p_s$$

Then,

$$\text{Prog}_E = \sum\nolimits_{s \in t} \left( \frac{N_{ps}}{N_p} - \frac{N_{ns}}{N_n} \right) PPEXP_s = \sum_{s \in t} \left( \frac{N_{ps}}{N_p} - \frac{N_{ns}}{N_n} \right) (\alpha_0 + \text{Prog}_S \times p_s)$$

$$= \sum_{s \in t} \left( \alpha_0 \left( \frac{N_{ps}}{N_p} - \frac{N_{ns}}{N_n} \right) + \text{Prog}_S \times \left( \frac{N_{ps}}{N_p} - \frac{N_{ns}}{N_n} \right) p_s \right)$$

$$= \alpha_0 \sum_s \left( \frac{N_{ps}}{N_p} - \frac{N_{ns}}{N_n} \right) + \text{Prog}_S \sum_s \left( \frac{N_{ps}}{N_p} - \frac{N_{ns}}{N_n} \right) p_s = \text{Prog}_S \times \text{Seg}_E$$

(The last equality follows from $\sum_s \left(\frac{N_{ps}}{N_p} - \frac{N_{ns}}{N_n}\right) = 1 - 1 = 0$)

Proposition 1 relates the two progressivity measures under student-based budgeting. Of course, as we have discussed, school funding does not take the simple student-based budgeting form, and therefore this relationship will not hold exactly. However, insofar as budgeting outcomes are approximated by different rates of allocations based on student poverty, we should expect the exposure-based measure of progressivity to be reduced from the slope-based measure of progressivity (which we could think loosely as the "underlying policy"), in proportion to segregation.

### 2.3 Geographic Decomposition of School Finance Progressivity Measures

The two progressivity measures introduced earlier in this section can each be geographically decomposed. "Geographic decomposition" means that each progressivity measure can be decomposed into a sum of within- and between-district components. This property is equivalent to "additive organizational decomposability" in the segregation literature, which is often mentioned as a desirable criterion for segregation indices (Reardon and Firebaugh 2002). It enables us to assess the relative contributions of different levels of government to the overall progressivity and therefore is useful to identify the locus of progressivity. For instance, if state A has a very progressive system between districts (i.e., a very positive progressivity measure) whereas the state is regressive or weakly progressive within districts, this state may want to monitor the within-district resource allocation to improve its progressivity.

The decomposition of the exposure measure is conducted similar to the hierarchical decomposition of exposure measures of segregation (Reardon and Firebaugh 2002; Jang 2022). Specifically, letting $N_{fd}$ (or $N_{nd}$) be the number of FRPL (or non-FRPL) eligible students in district $d$, state-level progressivity $\text{Prog}_\text{E}$ can be decomposed into between-district progressivity

($\text{Prog}_{\text{E}_\text{D}}$) and a weighted average of within-district progressivity ($\text{Prog}_{\text{E}_\text{d}}$ for district $d$),

$$\text{Prog}_{\text{E}} = \text{Prog}_{\text{E}_\text{D}} + \sum\nolimits_{d \in s} w_d \text{Prog}_{\text{E}_d} \tag{2}$$

where

$$\text{Prog}_{\text{E}_D} = \sum\nolimits_d \left(\frac{N_{fd}}{N_f} - \frac{N_{nd}}{N_n}\right) PPEXP_d \,;\; P_d = \sum\nolimits_{s \in d} \left(\frac{N_{fs}}{N_{fd}} - \frac{N_{ns}}{N_{nd}}\right) PPEXP_s$$

$$w_d = \frac{\pi_d(1-\pi_d)N_d}{\pi(1-\pi)N}$$

The within-district component $\sum_{d \in s} w_d \text{Prog}_{\text{E}_d}$ is a sum of within-district progressivity $\text{Prog}_{\text{E}_d}$ in district $d$ weighted by $w_d = \frac{\pi_d(1-\pi_d)N_d}{\pi(1-\pi)N}$, a combination of within-district student diversity and district size. More diverse districts (measured by the variance of socioeconomic composition, i.e., $p_d(1-p_d)$, where $p_d$ is the proportion of FRPL eligible students in district $d$) and larger districts (i.e., larger $N_d$) get more weight. Note that $\text{Prog}_{\text{E}_\text{D}}$ above is the exposure-based progressivity we would calculate when we only have district-level spending data without school-level spending information. In other words, $\text{Prog}_{\text{E}_\text{D}}$ is the progressivity statistic to be calculated if school districts were assumed to spend equally across schools within their jurisdictions.

The decomposition of the slope-based progressivity is conducted using properties of ordinary least squares regression as follows. First, by definition, this progressivity measure is a bivariate ordinary least squares slope,

$$\text{Prog}_S = \frac{Cov(PPEXP_s, p_s)}{Var(p_s)}$$

Now consider predicting school-level poverty rates ($p_s$) using district fixed effects. Let the predicted values be $\hat{p}_s$. These are, for each school $s$, the percent poor in school $s$'s district.

Then,

$$p_s = \hat{p}_s + u_s$$

where $u_s$ is the residual, which is orthogonal to $\hat{p}_s$. Therefore,

$$Cov(PPEXP_s, p_s) = Cov(PPEXP_s, \hat{p}_s) + Cov(PPEXP_s, u_s)$$

and

$$Var(p_s) = Var(\hat{p}_s) + Var(u_s)$$

Thus,

$$\begin{aligned}
\text{Prog}_S &= \frac{Cov(PPEXP_s, p_s)}{Var(p_s)} = \frac{Cov(PPEXP_s, \hat{p}_s)}{Var(p_s)} + \frac{Cov(PPEXP_s, u_s)}{Var(p_s)} \\
&= \frac{Cov(PPEXP_s, \hat{p}_s)Var(\hat{p}_s)}{Var(p_s)Var(\hat{p}_s)} + \frac{Cov(PPEXP_s, u_s)Var(u_s)}{Var(p_s)Var(u_s)} \\
&= \frac{Cov(PPEXP_s, \hat{p}_s)}{Var(\hat{p}_s)}\left(\frac{Var(\hat{p}_s)}{Var(p_s)}\right) + \frac{Cov(b_s, u_s)}{Var(u_s)}\left(\frac{Var(u_s)}{Var(p_s)}\right) \\
&= \frac{Cov(PPEXP_s, \hat{p}_s)}{Var(\hat{p}_s)}\left(\frac{Var(\hat{p}_s)}{Var(p_s)}\right) + \frac{Cov(PPEXP_s, u_s)}{Var(u_s)}\left(1 - \frac{Var(\hat{p}_s)}{Var(p_s)}\right) \\
&= R^2 \times \text{Prog}_S^{\text{Between}} + (1 - R^2) \times \text{Prog}_S^{\text{Within}}
\end{aligned}$$

where $\text{Prog}_S^{\text{Between}}$ is the slope to be obtained if we used only school pending and share poor aggregated at the district level, $\text{Prog}_S^{\text{Within}}$ is the slope to be obtained if we removed district effects, and $R^2$ is the percent of the variance in poverty explained by district fixed effects. The between-district contribution we take thus to be $R^2 \times \text{Prog}_S^{\text{Between}}$, while the within-district contribution we take to be $(1 - R^2) \times \text{Prog}_S^{\text{Within}}$. These sum up to the total progressivity slope.

## 3 Theory of Within-School Resource Allocation

The key distinction between the two measures lies in their perspective on the within-

school resource allocation process, in other words, how schools allocate resources across students by poverty status given their budgets from higher-level governments (e.g., district and state government). Specifically, the two measures are justified by two particular cases of within-school allocation: *within-school equality*, where the poor and non-poor students receive the same amount of investment once they attend the same school, and *within-school progressivity alignment* with the higher-level government, where schools mimic higher-level governments' progressivity in within-school resource allocations by assigning the same weights on poor and non-poor students as used by districts or states.

*Within-school equality* reflects an important strand of the literature in desegregation research (Johnson 2011; 2019). The idea is that access to school-level resources may not be practically stratified across students within schools. Or, even if schools do not equally distribute observed resources across students, it can still be considered close to equal distribution if we believe that substantial spillover effects occur among students within the same school. In this case, a dollar spent at a school is best thought of as spread equally among all students within the school.[1]

The exposure measure is an exactly accurate measure of progressivity under *within-school equality*. In this case, the fact that the exposure measure is expected to be systematically related to segregation (Proposition 1) is not a flaw in the measure, rather, it reflects reality. When a dollar spent at a school is spread equally among all students, then schools with high concentrations of poor students do indeed have higher per-pupil spending under a progressive student-based budgeting formula. Consequently, the exposure-based progressivity measure would imply this "silver lining of segregation." On the other hand, if the budgeting formula assigns fewer resources to poor students than non-poor students, segregation will amplify this

regressivity.

By contrast, *within-school alignment* assumes that schools follow the same level of progressivity in their within-school resource allocation by stratifying resources across poor and non-poor students based on their state and district's student-based budgeting scheme. For instance, they may offer special programs for English learners or students with disabilities, for which poor students could be disproportionately eligible (Mancilla-Martinez et al. 2022). If schools prioritize such programs that can benefit poor students more than their non-poor peers, the within-school resource allocation could be comparable to the across-school allocation.

If *within-school alignment* is accurate, the slope measure is a more appropriate description of funding progressivity since it is based on an individual student's characteristics, regardless of their school-level demographics. If this assumption holds, the funding progressivity is invariant to aggregation (Observation 2). In this case, whether using district-, school-, or even classroom-level fiscal data, the slope measure will produce the same value on funding progressivity. However, if we suspect substantially high resource spillovers across students within schools, then the impact of within-school targeting would be diluted and make the slope measure an inaccurate description of funding progressivity.

Given limited research understanding of how educational spending is allocated within schools, we do not take a stand on whether there a clearly right or wrong answer how to view the within-school resource allocation. However, we argue that, based on our reading of the literature and despite some arguments to the contrary, within-school equality is likely closer to what is happening in schools whereas the within-school alignment weighs more on the intended allocation of resources by district, state, and federal governments. This is because schools cannot accurately distinguish poor from non-poor students within their system unless they use the exact

label of student poverty in classroom assignment, which they do not. Moreover, as far as poor and non-poor students share the same social space (i.e., school), the benefits of educational investment may be shared given the nature of spillover effects (de Heer et al. 2011). Therefore, while the slope-based measure can be understood as the underlying policy parameters, the exposure-based measure may better capture *de facto* progressivity experienced by students and therefore is more intuitive.

In addition to these two viewpoints, one may argue that within-school allocations are neither equal across students nor aligned with the allocations of higher-level governments. For instance, a within-school *opportunity hoarding* theory suggests that schools are (perhaps far) more regressive than higher-level governments in their budgeting. For instance, within-school student sorting may assign more novice teachers to classrooms concentrated with racially minoritized or poor students (Kalogrides and Loeb 2013). More experienced teachers may teach the advanced courses taken mostly by non-poor students. In this case, even if districts and states allocate more money to schools with higher poverty rates, poor students may not benefit from funding as intended, thereby making progressivity—regardless of which measure is used—overestimated.

This last story highlights that to interpret progressivity measures using across-school data, assumptions about how investments are allocated within-schools are necessary. We now turn to formal arguments using the assumptions we have presented verbally in this section to justify the two progressivity measures we study.

### 3.1 Formal Proofs of Conditions Under Which Each Measure is Exactly Accurate

In this subsection, we provide mathematical statements and proofs of the conditions under which the exposure-based and slope-based progressivity measures are exactly accurate

descriptions of progressivity. Consider a collection of schools using student-based budgeting based on student poverty. Each school $s$ has a within-school allocation formula across students $i$ (this is the school-level policy)

$$E_i = \gamma_{0s} + \gamma_{1s} P_i$$

where $E_i$ is spending on student $i$ and $P_i$ is an indicator of student $i$ being from a poor family. The school policy parameters $\gamma_{0s}$ and $\gamma_{1s}$, which capture how the school *de facto* makes educational investments across students by type, can vary across schools $s$: under this policy, the amount of spending for poor and non-poor students will be $\gamma_{0s} + \gamma_{1s}$ and $\gamma_{0s}$, respectively. Of course, this formula is probably mostly implicit, driven by the school's relative spending on programs such as advanced courses versus tutoring, extracurriculars versus special education services, etc.

Each school in the collection receives a budget following an across-school allocation formula,

$$\frac{E_s}{N_s} = \beta_0 + \beta_1 \frac{|P_s|}{N_s}$$

where $E_s$ is total school spending, $N_s$ is the number of students in the school, and $|P_s|$ is the number of poor students in the school, and $\beta_0, \beta_1$ are across-school allocation parameters. In particular, $\beta_1$ is the parameter governing student-based budgeting: a school with 1 percentage point more poor students is allocated $0.01 \times \beta_1$ dollars more funding per pupil. For later notation, let $P_s$ be the set of poor students in the school and $NP_s$ be the set of non-poor students in the school, and $|\cdot|$ denotes the cardinality of the set.

Prior to a choice of measure based on aggregate data, school spending *progressivity* is conceptually defined as the difference in the average expenditure (actual investment) between the poor and non-poor students ($Prog$),

$$Prog = \frac{\sum_{i \in P} E_i}{|P|} - \frac{\sum_{i \in NP} E_i}{|NP|}$$

Where $P$ and $NP$ are the sets of poor and non-poor students in the district. Manipulating this expression,

$$Prog = \frac{\sum_s \sum_{i \in P_s} E_i}{|P|} - \frac{\sum_s \sum_{i \in NP_s} E_i}{|NP|}$$

$$= \frac{\sum_s \sum_{i \in P_s} (\gamma_{0s} + \gamma_{1s})}{|P|} - \frac{\sum_s \sum_{i \in NP_s} \gamma_{0s}}{|NP|}$$

$$= \frac{\sum_s \sum_{i \in P_s} \gamma_{0s}}{|P|} - \frac{\sum_s \sum_{i \in NP_s} \gamma_{0s}}{|NP|} + \frac{\sum_s \sum_{i \in P_s} \gamma_{1s}}{|P|}$$

The schools take the across-school policy parameters $\beta_0$ and $\beta_1$ as given and choose $\gamma_{0s}$ and $\gamma_{1s}$. We assume that the poor and non-poor students are immobile, so all governments take their locations (which schools they are in) as given. Each school, therefore, faces a school-level budget constraint in which the sum across the school's student-specific spending must equal the school's allocated budget, which can be expressed as

$$\frac{\sum_{i \in s} E_i}{N_s} = \frac{\sum_{i \in s} (\gamma_{0s} + \gamma_{1s} P_i)}{N_s} = \gamma_{0s} + \gamma_{1s} \frac{|P_s|}{N_s} = \beta_0 + \beta_1 \frac{|P_s|}{N_s}$$

Because a school cannot invest a negative amount into a student, we have the constraints $\gamma_{0s} \geq 0$ and $\gamma_{0s} + \gamma_{1s} \geq 0$. Given these constraints, the school still has many policy options, including investing everything in poor students ($\gamma_{0s} = 0$) or everything in non-poor students ($\gamma_{0s} + \gamma_{1s} = 0$). Consequently, schools could undo whatever aggregate-level progressivity we observe, which makes it unclear what the "true" progressivity is unless the school expenditure data is collected from individual students. (Except for the particular case of schools that have no poor or non-poor students, in which the school only invests in one student type.) Therefore, to provide any measure of progressivity using aggregate data, we are forced to make assumptions

about what happens inside schools.

We now consider two special cases, corresponding to the two progressivity measures delineated in this paper.

*a. Within-school equality*

Student's poverty status does not affect the amount of school resources she/he receives (i.e., $\gamma_{1s} = 0$) and the above expression simplifies to

$$Prog = \frac{\sum_s \sum_{i \in P_s} \gamma_{0s}}{|P|} - \frac{\sum_s \sum_{i \in NP_s} \gamma_{0s}}{|NP|}$$

because of the school-level budget constraint, $\gamma_{0s} = \beta_0 + \beta_1 \frac{|P_s|}{N_s}$, and thus

$$Prog = \frac{\sum_s \sum_{i \in P_s} \left( \beta_0 + \beta_1 \frac{|P_s|}{N_s} \right)}{|P|} - \frac{\sum_s \sum_{i \in NP_s} \left( \beta_0 + \beta_1 \frac{|P_s|}{N_s} \right)}{|NP|}$$

$$= \beta_1 \left( \frac{\sum_s \sum_{i \in P_s} \left( \frac{|P_s|}{N_s} \right)}{|P|} - \frac{\sum_s \sum_{i \in NP_s} \left( \frac{|P_s|}{N_s} \right)}{|NP|} \right) = Prog_S \times Seg_E = Prog_E$$

*b. Within-school alignment*

Schools allocate their resources following the across-school policy parameters for resource allocation across student poverty (i.e., $\gamma_{0s} = \beta_0$ and $\gamma_{1s} = \beta_1$), and the above expression simplifies to

$$Prog = \frac{\sum_s \sum_{i \in P_s} \beta_0}{|P|} - \frac{\sum_s \sum_{i \in NP_s} \beta_0}{|NP|} + \frac{\sum_s \sum_{i \in P_s} \beta_1}{|P|}$$

$$= \frac{\sum_s \sum_{i \in P_s} \beta_1}{|P|} = \beta_1 = Prog_S$$

Together these results prove our claims.

## 4 Data

For our study, we combine the National Education Resource Database on Schools (NERD$) data for 2018-19 with the same school year's Longitudinal School Demographic Dataset (LSDD), a cleaned version of the Common Core of Data (CCD). Table 1 provides summary statistics on the variables we use from these sources. Columns (1) through (5) provide summary statistics for our analytic sample, i.e., the data we use to measure progressivity. Columns (6) and (7) provide particular statistics for the original NERD$ data.

The original school-level finance data from NERD$ contains information on schools in all states except South Dakota. We make the following cuts in extracting our analytic sample from this original data. First, we noticed that the original NERD$ data has extreme outliers in spending (see column (6) of Table 1). We thus remove schools from our analysis that reported more than $50,000 spending per pupil. Second, we remove schools with nonpositive reported spending and schools with zero reported enrollment. Ultimately, our analytic sample includes 90,213 schools in 50 states (inclusive of D.C.; note that South Dakota is missing from the data).

Of these restrictions, the most consequential is our removal of outlier schools based on spending. Our progressivity measures, which are based on means, may be sensitive to the choice of restricting outliers. As shown in Table 1, column (6), there are schools with over 10 million dollars of per-pupil spending, and these schools, if included, would drive the progressivity measures we report.[2]

The NERD$ data contains two measures of school spending: "normed" spending which is designed to make spending data comparable across states, and "raw" spending which is as reported by the state education agency (SEA). In this paper, we use the raw spending data. Our decision is driven by concerns of data loss: school-level normed spending is only provided when

the district-level sum of NERD$ school expenditure matches with the district-level spending in the Census Bureau's Annual Survey of School System Finances (F-33), which is unfortunately not the case in many large districts. Due to this, if we used the normed spending data, we would lose Los Angeles Unified and Mesa Unified, the largest school districts in California and Arizona, respectively, and for some states we would lose a substantial amount of data (Iowa loses 89% of schools, Vermont 82%, Kentucky 50%, California 46%, Oregon 34%, New Jersey 33%, Arizona 33%).

The downside to using raw spending data is the lack of comparability across states: states count student enrollment differently when computing per-pupil spending (e.g., some states use attendance measures such as average total daily attendance, other states use enrollments on a specific date), and states vary over to what extent they include spending such as transportation and food services in their school spending. Using raw data therefore may bias our estimates. However, it is impossible to gauge whether the bias due to data loss is more or less severe than the bias due to using raw spending data, since there is no ground truth.[3] But given the sheer amount of data loss if we used normed data, we have decided in this paper to use the raw measures. We encourage future research to return to this issue.

LSDD provides a cleaned version of CCD by the National Center for Education Statistics, which provides school-level enrollment aggregated by race/ethnicity and free/reduced-price lunch (FRPL) eligibility in all U.S. public schools. LSDD identifies data anomalies in CCD and imputes missing and implausible data. This feature is useful to address the 11 percent of the schools missing FRPL counts in 2018-19 and adjust FRPL counts that may be overreported due to the Community Eligibility Provision (CEP). For more information, see http://segindex.org.

## 5 Estimates of School Spending Progressivity and its Decomposition

Figure 1 illustrates school spending progressivity in the entire nation when estimated with the exposure-based ($\text{Prog}_\text{E}$) and slope-based ($\text{Prog}_\text{S}$) measures. The exposure-based measure of progressivity is the difference in per pupil spending of an average poor student's school versus an average non-poor student's school, which corresponds to \$189 (=\$12,247 - \$12,058). This indicates that poor students, on average, attend schools spending \$189 more per pupil than their non-poor counterparts. On the other hand, the slope-based measure is the bivariate regression coefficient of per-pupil spending on share poor in the school, and suggests a school where all students are poor spends about \$671 per pupil more, on average, than a school with no poor students. Both measures are weighted by school enrollments.

We also include in Figure 1 bubbles representing the binned conditional means, for bin widths of 10 percentage points along the x-axis (and thus, ten bubbles are shown in total). We include these to provide a better view into the raw data, although we do not make use of them further in this paper. In brief, these binned conditional means show that the true conditional mean function relating per-pupil spending to share poverty in the school is quite nonlinear. While the slope-based progressivity measure does not directly reflect these nonlinearities, we believe studying this nonlinearity directly (especially insofar as it persists within states and places and affects people's views of equity) would be a useful exercise for future research.

Figure 2 illustrates the distribution of state-level school spending progressivity, and its decomposition into between- and within-district components for each state. As described in Section 2.3, both progressivity measures can be geographically decomposed. The vertical lines in each plot are national progressivity, identical to those displayed in Figure 1. Marked with "x"s in the figure, state-level spending progressivity has a considerable variation across the states. Most

of the states are progressive (44 out of 50) based on our measures of progressivity, indicating that low-income students go to schools receiving more public educational funding than their middle- or high-income peers, on average. Note that all states have both exposure- and slope-based progressivity measures of the same sign, but the rankings do not necessarily match. This is expected, since the two measures are different, and in particular they make different (implicit) assumptions about resource allocations within schools (Section 3).

Another important takeaway from Figure 2 is that the within-district component of progressivity matters in a fair number of states. Consider for instance, states like Florida or North Carolina, which have a few very large school districts. Using district-level spending data—which most previous studies do (Baker et al. 2021; Blagg, Gutierrez, and Terrones 2022)—would allow us to construct between-district progressivity only, which can mask the fact that within-district funding allocations in these states are meaningfully progressive compared to other states. That the within-district component of progressivity is important suggests that school-level finance data is valuable for assessing the relative distribution of resources across student types. Moreover, progressivity may look different if we considered classroom-level or student-level allocations versus school-level allocations. On the whole, different values in spending progressivity across different organizational levels imply that the level of aggregation matters.

Since both the exposure- and slope-based measures of progressivity has been frequently used in the policy debate, and often they are used to rank states (Baker et al. 2021; Blagg, Gutierrez, and Terrones 2022), it is worth evaluating to what extent the two measures provide meaningfully different state rankings. Figure 3 provides evidence speaking to this question, by relating the state rankings on slope-based progressivity (x-axis) with rankings on exposure-based progressivity (y-axis). In this figure, the state ranked 1 is the least progressive, and the state

ranked 50 is the most. Note that based on Proposition 1, we would expect rankings to be very different if levels of economic segregation were very different across states. There are differences in rankings, and sometimes these differences are substantial. However, the overall correlation between the two approaches to measuring state-level progressivity is quite high, with a (Spearman) $R^2$ about 71% when we compare the overall state-level progressivity measures (top panel). We also include a comparison for the between-district component of progressivity in the bottom panel of Figure 3. The between-district rankings are what would be obtained if we averaged our data at the district level, assuming districts allocated per-pupil spending across schools within-district evenly. The $R^2$ between the slope-based measure and the exposure-based measure is larger for the between-district rankings than for the total rankings; this is likely because there is more variation in segregation across states when it is measured at the school-level (which is relevant for the top panel) versus when it is measured at the district-level (relevant for the bottom panel).

Figure 4 relates the two measures across school districts, showing a strong positive relationship between the two measures. The figure also highlights the top 100 largest school districts by FRPL eligible student enrollment in the country (as white dots), representing 19% of all public school students in the country (23.2% of the entire public school FRPL eligible population; these are calculated by comparing enrollment sums from our data to 2018-19 enrollments from NCES, 2020). Two patterns about these large districts stand out from Figure 4. First, the white dots are mostly located on the first quadrant of the figure, where both exposure and slope measures of progressivity are positive, suggesting that large districts are progressive in school spending regardless of how it is measured. Second, the association between the two progressivity measures is weaker among these large districts ($R^2$=0.48) than among all other

districts ($R^2$=0.64). This may be attributed to the fact that large districts have more room to have meaningful variation in economic segregation, which is expected to lead to differences between the two measures of progressivity based on Proposition 1.

While these relationships are interesting, we do not think that the primary intellectual benefit from comparing these two measures is in examining the robustness of states or school district rankings to different measurement approaches. Instead, we argue that what we learn from thinking critically about these two measures is that they take distinct perspectives on how resource allocation occurs within schools, as we have discussed. Moreover, these theories—and therefore, the measures—imply important differences in the relationship between progressivity and segregation. We turn to an empirical evaluation of that relationship in the next section.

## 6 The Association between Funding Progressivity and Segregation

We derived in Section 2.2 a theoretical relationship connecting the two measures of progressivity via segregation. This theory, however, was derived under the simplified student-based budgeting model, which only partly reflects school funding distribution in practice. Moreover, segregation itself could be associated with other dimensions of school funding policy even if student-based budgeting were a reasonable approximation; for example, high level of segregation may be associated with a less progressive political climate. Therefore, to see how our theoretical models are connected to the real world, this section examines the empirical relationship between the two progressivity measures and economic segregation.

Figure 5 depicts the association between funding progressivity and segregation for our two measures of progressivity. In both cases, we use the exposure measure of segregation for poor students ($\text{Seg}_E$ in our earlier notation), as that is the relevant measure that links the two

progressivity measures (Proposition 1). Each row corresponds to the slope-based (top row) and exposure-based (bottom row) measure of progressivity. The left column presents the state-level relationship, where the measures of progressivity and segregation are calculated at the state level, aggregating across schools within each state. In the middle column, all measures are computed at the between-school, within-district level, aggregating across schools within each district. In the right column, we repeat the middle column except only for the top 100 largest districts in the country by FRPL enrollment.

Two notable patterns appear in Figure 5. First, the progressivity-segregation relationship is negative at the state level (left column), whereas it is positive at the district level (middle column). This directional difference implies that it is the within-state variation driving the creation of highly resourced schools with concentrated poverty. Second, across all columns, the exposure-based progressivity has a more positive (or less negative) relationship with segregation than the slope-based progressivity. This can be understood as a result of shrinkage from the slope- to the exposure-based progressivity by economic segregation, stated in Proposition 1. At the state level, the slope-based progressivity has a negative relationship with economic segregation, but this relationship becomes essentially flat for the exposure-based one. According to Proposition 1, the slope-based progressivity in highly segregated states tends to be shrunken more than their less segregated counterparts, which attenuates the negative progressivity-segregation relationship on the exposure measure. And this results in a more positive progressivity-segregation relationship on the exposure than slope measure for districts, where it already has a positive on the slope measure. Our findings are starker when we focus on the top 100 largest districts by FRPL only (right column). The exposure measure shows a highly positive relationship between segregation and progressivity, with an $R^2$ of about 47%, while this

relationship is essentially flat on the slope-based progressivity.

If we believe in *within-school equality* in resource allocations and trust the school finance data, then the district-level results (especially for the top 100 districts) suggest that more segregated districts allocate more money to poor students (bottom right plot in Figure 5). Segregation combined with progressive school finance policy will lead to resource-rich schools of poor students, and resource-poor schools of rich students, supporting the "separate but better" scenario. But is this true? Is this a "silver lining of segregation?" By concentrating the poor in their own schools, can we increase the average resources targeted to them? We further question this in the next subsection.

### 6.1 A Missing Piece: Parental Contribution in School Spending

The observed progressivity in this paper may overlook one meaningful piece in school resources: *private*, parental contributions. This is because school spending data often are limited to the *public* contributions of district, state, and federal governments. The unavailability of private contribution data is concerning as parental contributions may account for a nontrivial part of the entire school resources[4] and they would be unevenly distributed across schools attended by students of different economic status. As a result, the total budget of the school, accurately defined, is both based on government allocations for poor and non-poor students as well as the parental contribution gap across schools with different student economic compositions.

We therefore propose an approach to evaluate spending progressivity that considers parental investments at the school in addition to observable public spending. Let $\eta_n, \eta_p > 0$ be the resources that non-poor ($\eta_n$) and poor ($\eta_p$) parents provide to the school, respectively, on a per-student basis. Then school $s$'s budget is

$$EXP_s = a_f + \alpha_n N_{ns} + \alpha_p N_{ps} + \eta_n N_{ns} + \eta_p N_{ps}$$

In per-pupil terms,

$$PPEXP_s = \alpha_0 + \{(\alpha_p - \alpha_n) - (\eta_n - \eta_p)\}p_s$$

The slope-based progressivity $\alpha_p - \alpha_n$ computed with the per-pupil *public* spending must be adjusted by subtracting the parental resource provision gap $\eta_n - \eta_p > 0$. From this perspective, considering parental contributions demands that public spending progressivity be sufficiently high to overcome the unobserved private regressivity of parental contributions.

We do not provide a full accounting of the gaps in common school investment between non-poor and poor parents, which is out of the scope of this paper. However, we explore how the relationship between progressivity and segregation would change depending on our assumptions about the parental contribution gap. If this gap is assumed to be higher, non-poor students in more segregated districts would attend schools with more parental resources collected. This in turn would flatten the relationship between the exposure-based progressivity and economic school segregation.

Focusing on the top 100 largest school districts, we find that for about a $1,750 gap in per-pupil parental investments, the relationship between exposure-based progressivity and segregation would be reversed. Figure 6 illustrates this result for this and several other examples of the parental investment gap, $\eta_n - \eta_p$. Notice that the relationship is approximately flat for a parental contribution gap of $1,750 (bottom-left panel).

This analysis provides an additional insight on the value of the exposure measure. From the perspective of the exposure-based progressivity, the concentration of poor students into segregated schools will produce pockets of highly-funded, high-poverty, educational institutions. The observed progressivity in public spending may then suggest a “silver lining” to segregation, in which “separate but better” would be a new doctrine. However, the unobserved resources

raised disproportionately by wealthier parents may be more than enough to overcome the modest degree of government-funded progressivity. As in Figure 6, it is possible that observed government-funded progressivity masks true regressivity in actual school resources. If this is the case, segregation continues to have a pernicious effect on school resource equity, by amplifying a fundamentally regressive school resource system.

Whether and to what extent parental inputs reverse the sign of observed government progressivity is an open question for future researchers. We also recognize that it is a valid position for advocates to want more resources for the poor to overcome other forms of societal disadvantage, regardless of the parental contribution gap. For example, rich parents privately invest resources in their children, and advocates may want public resources to compensate for this. But segregation does not necessarily amplify such investment contrasts, since the nature of private investments suggests that rich parents in schools serving mostly poor students would privately spend on their children's education anyway. By contrast, if rich parents invest in the local public school (not in their children), segregation will directly amplify this resource inequity. If there are sufficient differences in how parents invest in their public schools based on their advantage, then there can remain inequity of schooling inputs even if the public financing of education "looks" progressive.

## 7 Discussion and Conclusion

With strong and tightening limits on desegregation policy, those interested in greater equity in education may consider turning to school finance to address the "unequal" component of the "separate but unequal" school system. Recent data collections suggest the possibility of a more transparent and accountable future for across-school resource allocations. There has been a

history of school finance equalization since the 1970s, and one of its impacts is the increased spending for more disadvantaged students (Rothbart 2020; Oberfield and Baker 2022) which improved low-income students' educational opportunities and social mobility (Hyman 2017; Lafortune, Rothstein, and Schanzenbach 2018). Moreover, the legal courts appear less opposed to, and in some cases directly supportive of, school finance reform policy (Rubenstein and Picus 1999; Hanushek and Lindseth 2009).

This policy context motivates a more careful discussion of the measurement of school spending progressivity. A number of policy reports and research papers provide a ranking of funding progressivity across states, many of which use one of the two measures we formalize, discuss, and compare in this paper (e.g., Baker et al. 2021; Blagg, Gutierrez, and Terrones 2022). While these approaches to measuring progressivity have been widely used, no study to date has provided a thorough review of them. To fill this gap, our present study overviews these different measures of funding progressivity, shows how they are related but distinct, and discusses how they rely on different assumptions about within-school resource allocations.

To this end, our study brings several contributions to the field of school finance. First, we formalize two popular approaches to measuring school finance progressivity: exposure-based and slope-based measures. The exposure-based measure of progressivity is the difference in school spending between the average poor and non-poor student's school. In contrast, the slope-based measure represents the linear association between school per-pupil spending and the share of poor in the school. Second, we study their theoretical relationship under the assumption of student-based budgeting. We show that, under this assumption, the exposure-based measure is shrunken from the slope-based measure in proportion to the exposure-based economic segregation. This shrinkage amplifies the relationship between the exposure-based measure and

economic school segregation. Finally, we contribute an empirical study of school finance progressivity across the U.S., including by-state decompositions of each progressivity measure into between- and within-district components, using new school-level spending data curated by NERD$. Our empirical analysis confirms the theoretical predictions.

We emphasize that each measure of progressivity is justified under different assumptions about the within-school allocation of public spending. The exposure-based measure is justified by *within-school equality*, in which students share the benefits of public spending once attending the same school. By contrast, the slope-based measure is justified by *within-school progressivity alignment*, where spending is allocated to poor and non-poor students differentially even in the same school, following the allocation scheme by federal, state, and district governments. Given the probable infeasibility of within-school resource targeting and the spillover effect of within-school resources, the assumption of within-school equality may be more plausible than within-school progressivity alignment to describe the actual resource allocation. We, therefore, suggest using the exposure-based progressivity to best reflect the true differences in public education spending for the disadvantaged relative to the advantaged. In any case, our discussion motivates researchers to better unpack how resources are allocated across students within schools.

While a better understanding of resource allocation within schools is undoubtedly important, we believe an even more relevant issue is the possibility of large and unobserved parental contribution gaps. We find that, empirically, there is a highly positive association among large school districts between segregation and exposure-based progressivity. This "silver lining of segregation" is predicted by our theory of within-school equality and exposure-based progressivity: separating poor from non-poor students would create highly-funded schools of concentrated poverty and relatively resource-poor schools of concentrated advantage, thereby

supporting the "separate but better" narrative. Yet, if non-poor parents privately contribute to their schools more than poor parents in the current segregated school system, these private contribution gaps could easily overturn the "separate but better" narrative, decreasing *de facto* resources in schools with concentrated disadvantage. We illustrate how a relatively modest gap in these unobserved parental contributions could reverse the observed "separate but better" pattern, returning us to the more intuitive "separate and worse" scenario. While we gave some rough estimates based on the literature that speaks to the possible size of this parental contribution gap, we leave careful documentation of these differences to future research. We believe a careful study of these gaps will be necessary to understand better how far the current U.S. education system is from equality of inputs, even apart from the more controversial issues of equity in outcomes or opportunity.

[1] The within-school equality condition is closely related to the literature on club goods in economic theory. The traditional model of clubs as written in Buchanan (1965), sometimes called the "Buchanan club" (Sandler and Tschirhart 1980), assumes that members of the club equally share of the impure (congestable) public good, and non-members get none of it (this is also assumed in McGuire 1974). This "equal utilization by all members" is exactly the within-school equality condition under which the exposure-based measure is accurate. Because the club goods literature is varied and complex, and these assumptions are not standardized, we do not refer to "within-school equality" as an assumption that schools are "club goods."

[2] We decided not to set a cutoff for schools with very low spending per-pupil, since even schools at zero dollars of reported spending per pupil are not pivotal outliers relative to the average.

[3] One exercise we conducted was to compare state-level progressivity estimates using raw versus normed data, restricting throughout to only schools that have normed estimates available. We found that overall national estimates of progressivity were very little changed, only about 1%, by norming. But switching from raw to normed data did change the progressivity estimates substantially in some states. The largest relative changes (greater than 33% in absolute value) were VA (-103% change from raw to normed), NY (+96%), NJ (-86%), VT (+73%), RI (-69%), NH (+54%), IA (-47%), IL (+45%), and MO (-35%).

[4] While the private contribution to district-level revenues is about 2% of total annual spending on public schools (about $240 per pupil; NCES 2021), schools receive other private resources through parent-teacher associations (Murray et al. 2019) or private donor groups in charter schools (Baker 2018, 68–72).

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**Tables and Figures**

Table 1: Summary Statistics.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| | **Analytic Sample** | | | | | **Original Data** | |
| **Variable** | **Mean** | **SD** | **Min** | **Max** | **N** | **Max** | **N** |
| School Spending Per-Pupil ($) | 13,009 | 5,489 | 0 | 49,957 | 90,213 | 18.9m | 91,463 |
| Percent Free/Reduced-Price Lunch (FRPL) (%) | 55 | 26 | 0 | 100 | 90,213 | 100 | 94,043 |
| Total Enrollment (Students) | 547 | 450 | 1 | 14,306 | 90,213 | 14,306 | 94,043 |

Notes: Table displays selected summary statistics for the original data before restrictions (columns 6 and 7), and more summary statistics for our analytic sample (columns 1 through 5). Our analytic sample restricts to schools with nonmissing data on school spending, free/reduced-price lunch, and enrollment, and less than $50,000 spending per-pupil.

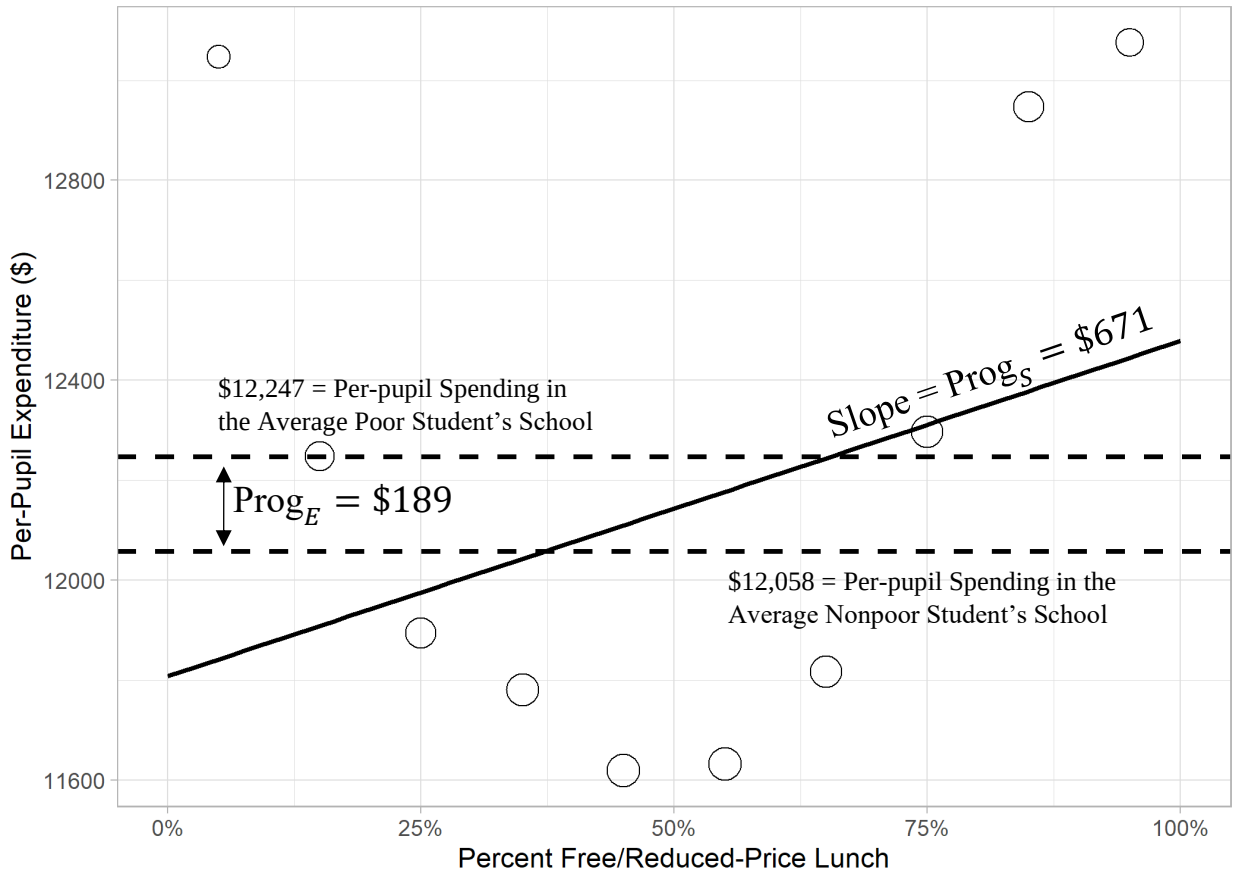


Figure 1: *National Progressivity Estimates for the Exposure-Based and Slope-Based Measures.* Notes: Figure illustrates the national value of the two progressivity measures, exposure-based $\text{Prog}_E$ and slope-based $\text{Prog}_S$. The slope-based progressivity measure is the coefficient on a regression of per-pupil expenditure on share free/reduced-price lunch (weighted by school enrollment). The exposure-based progressivity measure is the difference between the weighted mean per-pupil expenditure for poor students minus non-poor students, as illustrated. The bubbles in the figure are binned averages, where the bins are width 10 percentage points and the size of the bubble is proportional to the total student enrollment in the bin.

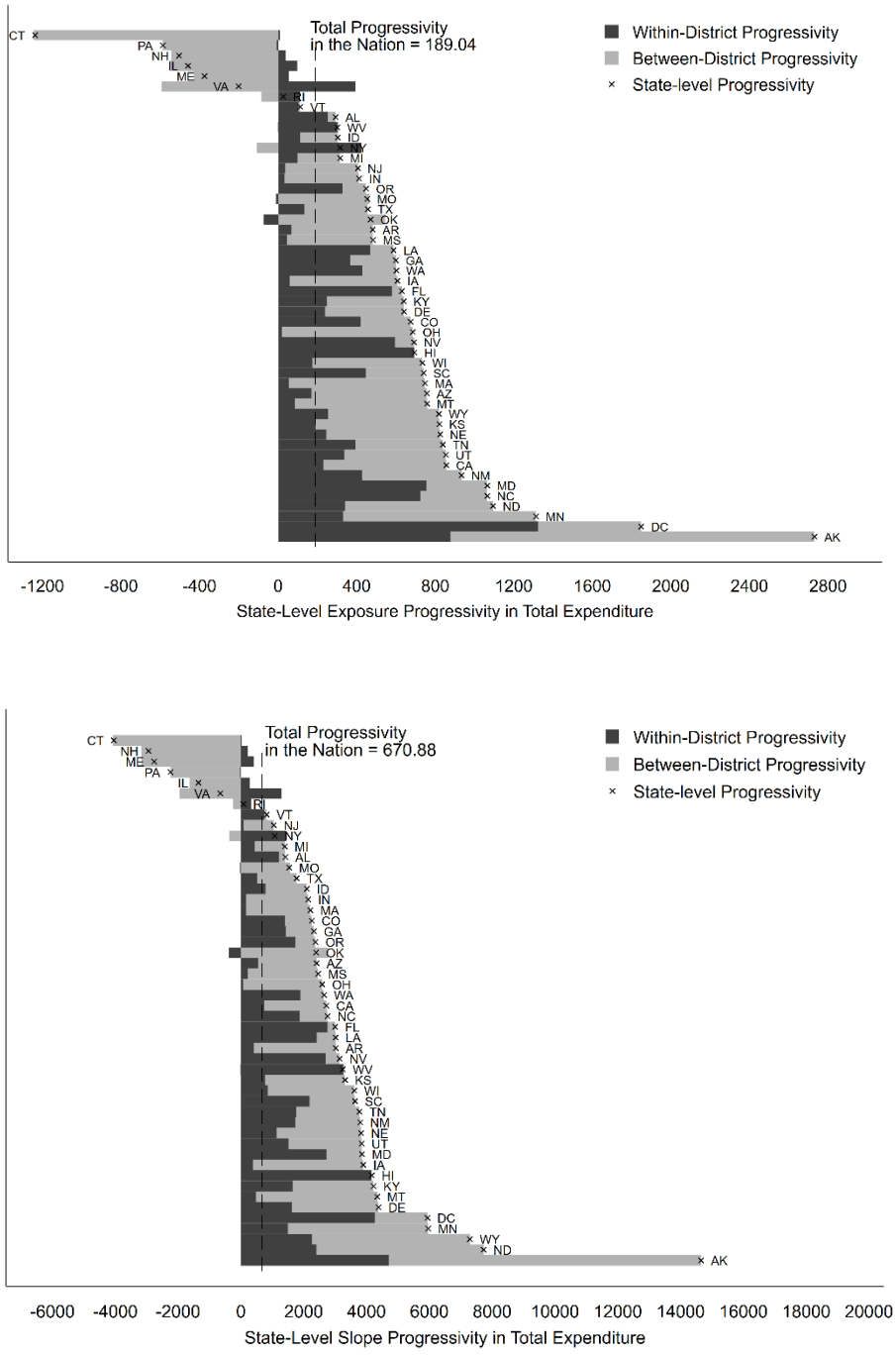


Figure 2: *The Distribution and Components of Progressivity Across U.S. States.* Figure displays total state progressivity (as x's), and the within-district and between-district components of progressivity (shaded bars) for exposure-based progressivity (top figure) and slope-based progressivity (bottom figure). National progressivity is displayed as a vertical dashed line. Data is restricted to the analytic sample.

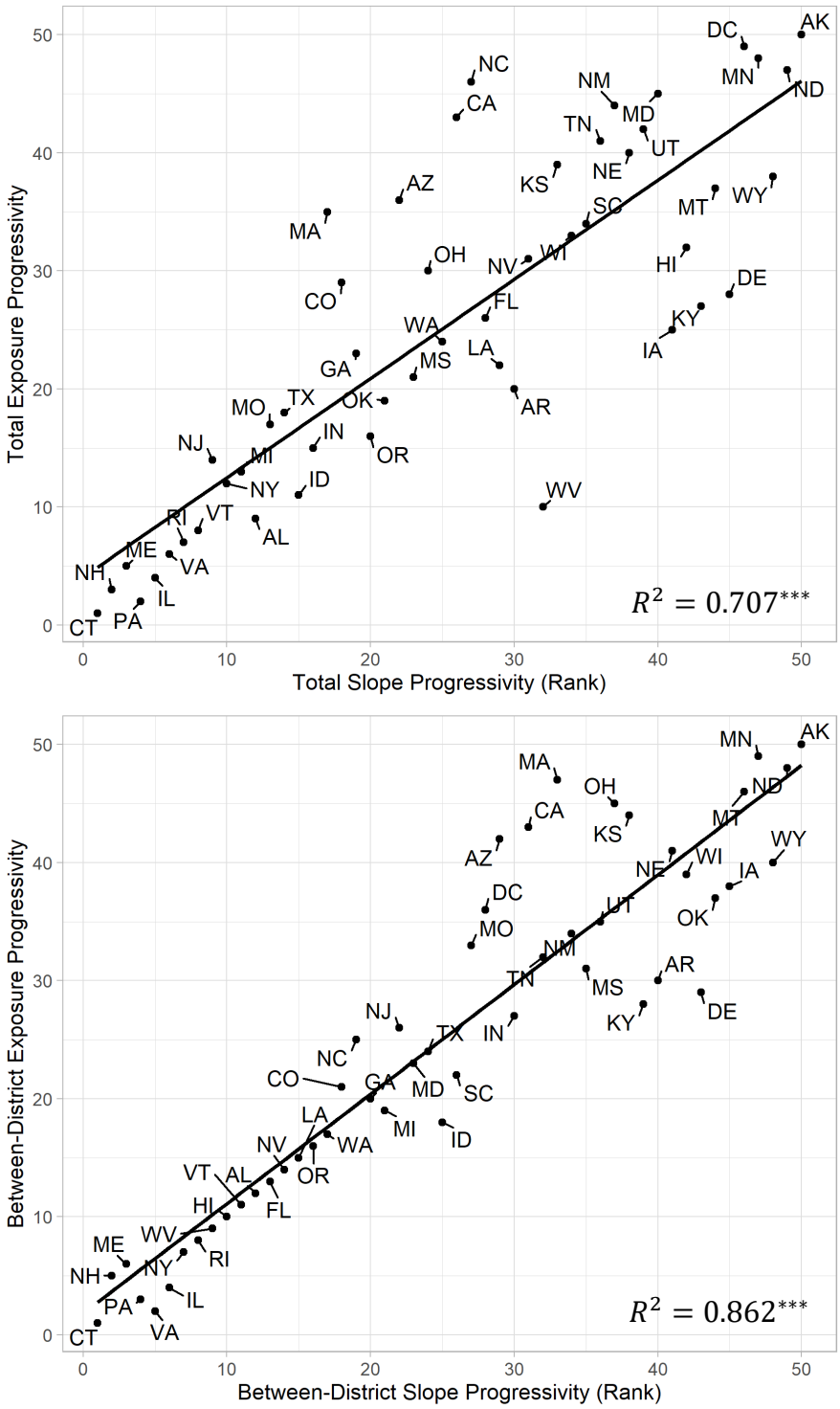


Figure 3: *Slope- Versus Exposure-Based Progressivity Across States.* Figures relates the state rank of slope-based progressivity (x-axis) versus the state rank of exposure-based progressivity (y-axis); the top figure uses the total progressivity measures, while the bottom figure uses between-district progressivity. Figures include linear regression lines of best fit, and $R^2$.

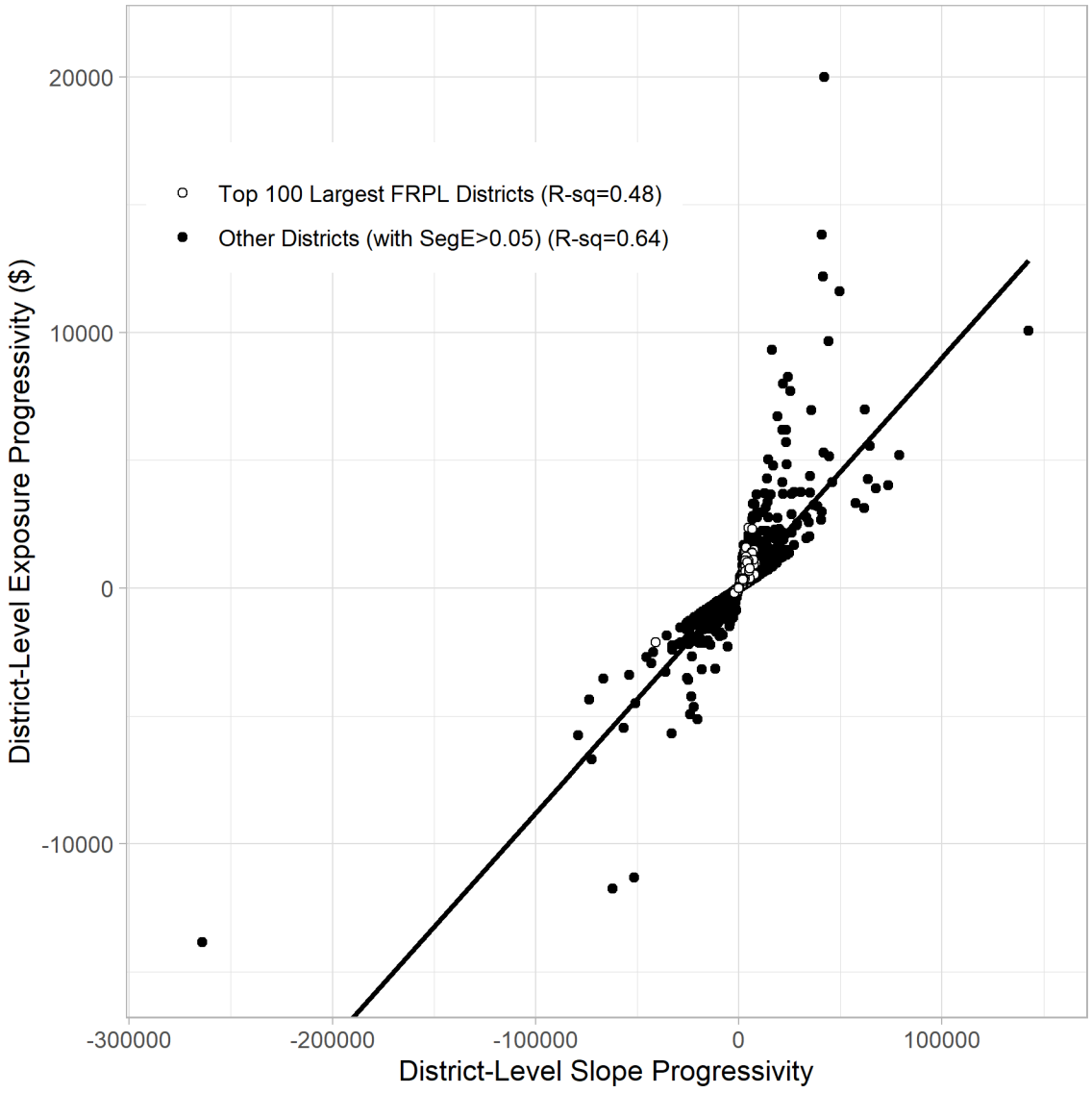


Figure 4: *Slope- Versus Exposure-Based Progressivity Across School Districts.* Figures relates district-level slope-based progressivity (x-axis) versus exposure-based progressivity (y-axis). The top 100 largest free/reduced-price lunch (FRPL) school districts are highlighted in white. Other districts displayed include only districts with at least 0.05 exposure-based segregation ($\text{Seg}_E > 0.05$; this condition implicitly requires the school district to have at least two schools). Figure includes a linear regression line of best fit for all districts.

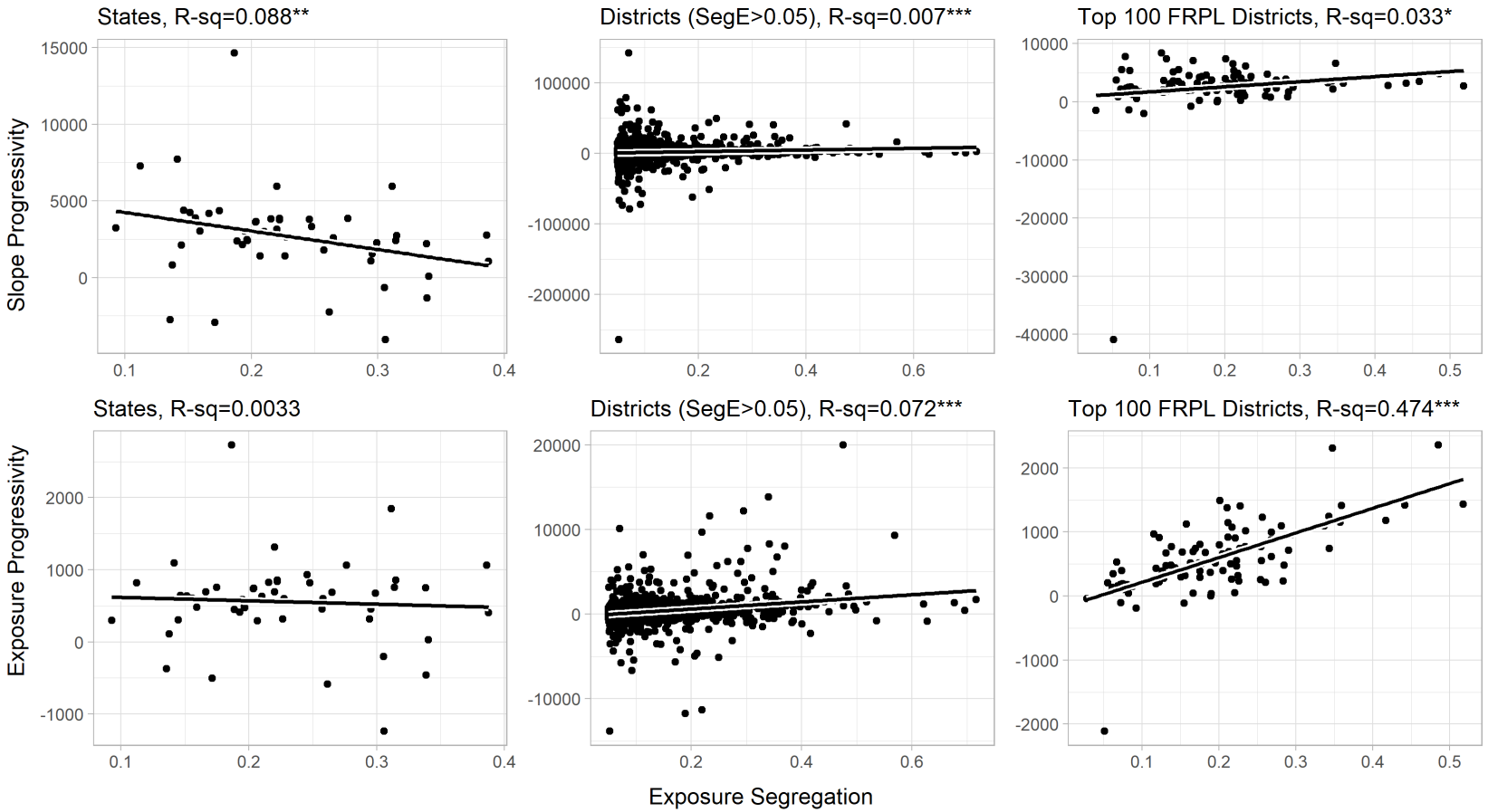


Figure 5: *The Association between Progressivity and Segregation across States and Districts.* Figure displays the relationship between exposure segregation ($\text{Seg}_E$, x-axis), and each progressivity measure (y-axis), across states, districts, or top 100 largest free/reduced-price lunch (FRPL) districts (across columns, respectively). Note that the y-axis scale freely varies across plots. Plot titles include $R^2$ estimates, with stars denoting significance level on a hypothesis test of an $R^2$ of zero (*$p < 0.10$, **$p < 0.05$, ***$p < 0.01$).

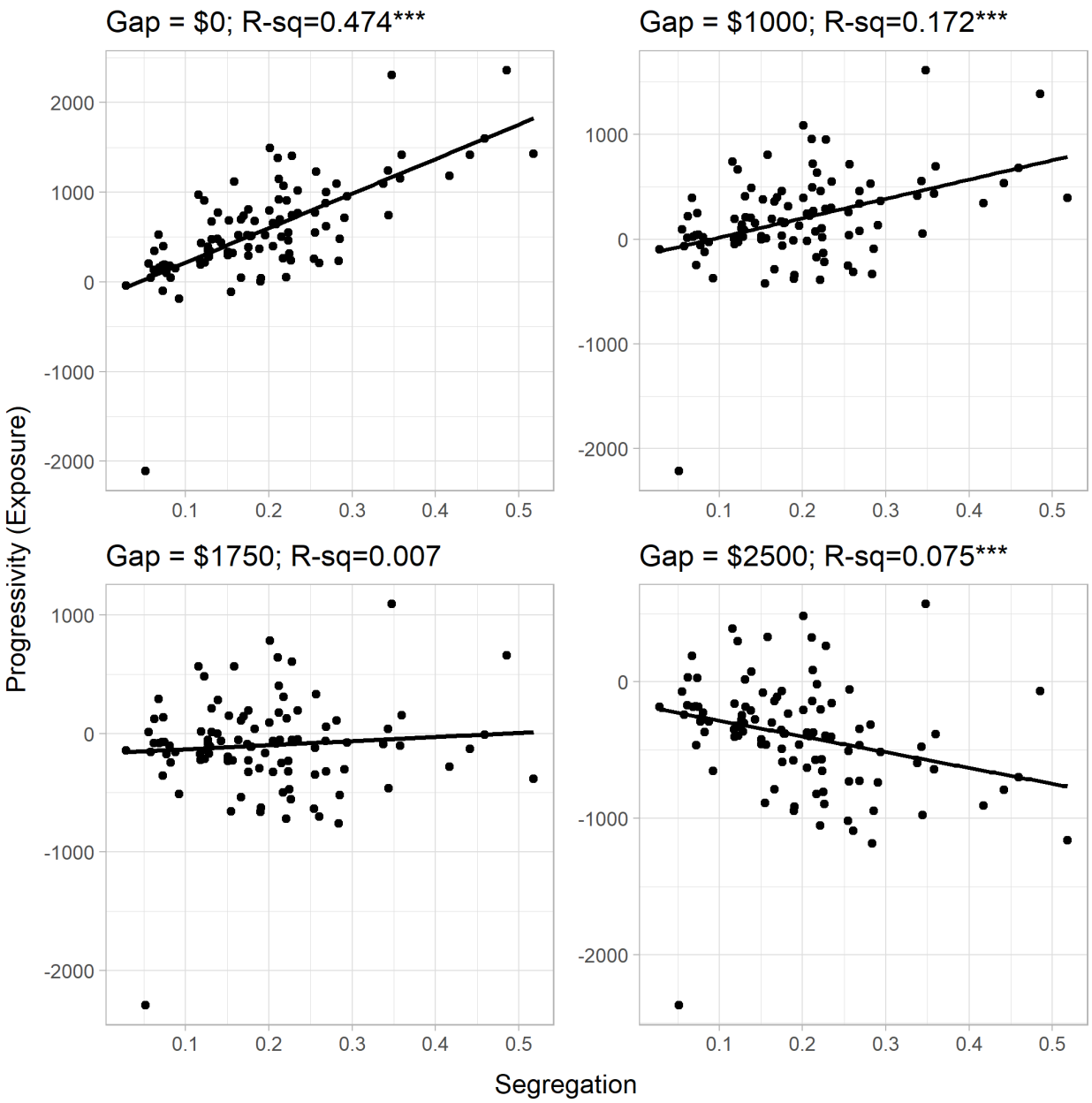


Figure 6: *Segregation and Progressivity (Exposure Measure) Across Top 100 Largest FRPL School Districts, Varying the Parental Contribution Gap*. The top left panel of this figure is identical to the bottom right panel of Figure 5. Other panels of this figure recalculate district-level exposure-based progressivity (y-axis) under different assumptions about the parental contribution gap, i.e. how much more unobserved resources non-poor parents bring to the school relative to poor parents. Plot titles include the assumed parental contribution gap, and $R^2$ estimates with stars denoting significance level on a hypothesis test of an $R^2$ of zero (*$p < 0.10$, **$p < 0.05$, ***$p < 0.01$).